

Drug lords, rebel movements and anti-drug policies in source countries*

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Abstract

U.S. strategy against the production and distribution of illegal drugs in and from source countries uses both supply and demand side anti-drug policies with the aim of increasing drug prices. In source countries, the trends for potential drug production (or area cultivated with coca and opium poppy leaves) and trafficking activities, together with wholesale prices for cocaine (and heroin) in the U.S. show no clear results supporting the success of the strategy. Moreover, the existence of these illegal industries has been an important factor in the development of rebel movements. This paper presents a possible explanation for the correlations between both anti-drug policies and these trends, by analyzing the illicit drug production and distribution together with the existence of rebel movements. By accounting for the interaction between a rebel movement, a drug lord, and a government, is possible to explain the effects of each of these two anti-drug policies, their relative effectiveness, and the reasons behind their use. The analysis suggests that demand oriented anti-drug policies can produce better results than supply oriented policies.

JEL codes: D78, K42, L20.

Keywords: illegal behavior, conflict, law enforcement.

*The presence of both rebel movements and drug lords are mainly found in the Andean and Golden Triangle regions. However, in the Andean region, governments, drug lords and rebel movements are different players. Hence, this paper applies specifically to the Andean region.

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1 Introduction

At least for the last two decades, drug lords have been selling "natural"¹ illegal drugs (e.g., cocaine and heroin) to final destination countries (e.g., the U.S. and the European countries). They have done it by growing these drugs in source countries (e.g., the Andean countries), cooperating with rebel movements², and hiring drug dealers to distribute them. Therefore, it is believed that by fighting against this illicit industry, drug production, drug trafficking, and rebel movements' strength will decrease.

According to the U.S. Drug Control Strategy report for the year 2002, the two main drug problems (in terms of treatment demand) faced by U.S. authorities have been heroin and cocaine use. For this country, meeting the challenge of reducing illegal drug use has required two important strategies: to disrupt both domestic and international markets and to invest in potential and actual drug users.

The first strategy has been used to identify drug production in source countries, destroy it, intercept what is left and sent to the U.S. market, and dismantle the drug networks that transport, distribute and sell the drugs throughout the U.S.. The main objective with this strategy is to drive up costs in the drug industry which would be passed on to final drug consumers who ultimately decrease use.

The second strategy has been to prevent drug use before it starts and get treatment resources where they are needed. The main purpose with this strategy is to decrease directly the demand for drugs.

For these "natural" drugs, the above strategies combine both supply and demand side anti-drug policies. Demand side policies refer to treatment and prevention programs, and the dismantling of drug networks that transport and distribute the drugs, which reduce the size of the drug market (decreasing drug price). On the other hand, supply side policies refer to involuntary crop eradication, destruction of clandestine labs, and interception of drug shipments, which decrease drug availability in the U.S. market (increasing drug price)³.

¹Drugs that are grown to be produced.

²The two main rebel movements in the Andean region ("*The FARC*" in Colombia and "*The Shining Path*" in Peru) are based in the territories where coca and opium poppy leaves are grown. According to U.S., Colombian and Peruvian authorities, during the past two decades, both groups have been actively involved in the drug industry mainly through the protection of crop fields and clandestine labs (see Kay (1999), United States General Accounting Office (1998), and Ministry of National Defense, Colombia (2000)).

³The distinction in the literature has not been very precise as some supply oriented anti-drug policies (e.g., interdiction programs) may affect not only the availability of drugs into the U.S. market but also the drug networks that transport and distribute the drugs.

Annually, millions of dollars are spent on spraying coca and opium poppy fields with herbicides; military operations to destroy drug laboratories; military infrastructure, radars, communication systems, etc., to intercept illegal drug shipments; law enforcement programs like the Organized Crime Drug Enforcement Task Force created in 1982 to focus resources on dismantling and disrupting major drug-trafficking organizations; and prevention and treatment programs to focus directly on the drug user⁴ (Figure A1).

However, there is little empirical evidence supporting the efficacy of these anti-drug policies on source countries. On one hand, cocaine and heroin wholesale prices have been constantly dropping in the U.S. market (Figure A2). On the other hand, in the Andean region as a whole there is no correlation between crop eradication and the territory cultivated with coca leaves (for Colombia the correlation is positive while it is negative for Peru) and/or opium poppy leaves (in Colombia), nor there is evidence that interception of drug shipments, has succeeded in either reducing drug availability or drug trafficking⁵ (Figures A3, A4 and A5). Moreover, for the past two decades the number of rebels in the region has increased⁶ with a higher growth rate during the 1980's (150% from 1986 to 1991) than during the 1990's (50% from 1991 to 2000) for the FARC movement (Figure A6).

While these dismal results could merely reflect changes in drug users' preferences (or other variables) in the final destination countries, neutralizing the effects of the policies, there are also theoretical reasons for why the policies applied in source countries may fail to produce the desired effects. These are the concerns of the present paper.

Specifically, this paper argues that is crucial to take into account the inter-

This last effect may have consequences on the market size as the distribution channels uncovered cannot be used in the future for supplying the drug. Therefore, the demand is suddenly disconnected from the supply and for traffickers this can be seen as a decrease in drug demand resulting in the dropping of drug price, a demand oriented anti-drug policy effect from an assumed supply oriented policy.

⁴See, United Nations Office of Drugs and Crime (UNDCO), Global Illicit Drug Trade, 1999-2003; United States General Accounting Office, "Drug Control: US Drug Interdiction Issues in Latin America", October, 1994; and The White House, National Drug Control Strategy 2002.

⁵Drug seizures and the number of trafficking routes towards the U.S. market have increased. See United Nations Office of Drugs and Crime (UNDCO), Global Illicit Drug Trade, 1999-2003.

⁶Due to the increase in the Colombian rebel movement which has been the biggest group in the region. It is important to mention that in both countries there is a clear correlation between drug production and the strength of the rebel movements. In Colombia, the data shows a positive correlation between illegal drug production and the strength of its main rebel movement, whereas in Peru, the peak and fall of the coca leaf production shows similar patterns to the dynamics of its main rebel movement (Figure A7).

action between two different players in the illicit industry in source countries in order to understand the unsuccessful results of the anti-drug policies. It models the interaction between a drug lord and a rebel movement, and a government,⁷ instead of assuming that supply oriented anti-drug policies will increase drug prices and decrease total drug production available in final destination countries, and demand oriented policies will directly decline this production.

By accounting for this interaction, the paper explains how an increase in supply oriented anti-drug policies could contribute to increase both the potential drug production⁸ and rebel spending without affecting trafficking activities, drug prices, and total drug production⁹. Supply oriented anti-drug policies do not affect total drug production because an increase of these policies decreases the drug productivity per acreage making the drug lord to increase the potential drug production. This increase offsets the decrease in productivity letting the total drug production in final destination countries unaffected¹⁰.

Moreover, intensification in demand oriented anti-drug policies could decrease the potential drug production, rebel spending, and total drug production, while having no effect on drug prices. These policies do not affect drug prices because their increase will be offset by the decrease in both the total drug production and the market size, in spite of the partial increase in distribution networks.

In a broader literature this paper is related to the conflict and appropriation theory leaded by Grossman (1991), Skaperdas (1992, 1996) and Hirshleifer (1991, 1995)¹¹. Moreover, since this paper is based on the idea that strategic interactions between players in the drug industry may have caused the failure of anti-drug policies, this paper is related to Skott and Jepsen (2002). However, they focus on addiction, imperfect competition, and the presence of switching costs and consumer loyalty, where anti-drug policies

⁷In the Golden Triangle, governments can be perceived as either drug lords or rebel movements and/or rebel movements can be perceived as drug lords. With respect to Afghanistan (main producer of opium poppy and heroin in Asia), the Taleban regime represented a government with very similar objectives to the ones from drug lords.

⁸In the empirical literature is common place to relate the potential drug production as a proportion of the territory cultivated with the drug. Therefore, both terms will be used indistinctly.

⁹Drugs sold or total drug production are all the same in equilibrium. Therefore, the terms are used indistinctly in this paper.

¹⁰As it will be discussed later on in the paper, this result comes from the fact that the marginal cost of the potential drug production is proportional to the productivity per acreage.

¹¹See Neary (1997) for a comparison between rent seeking and conflict models.

are assumed to affect cost parameters and are not explicitly modeled.

More specifically, there are very few papers examining the interaction between drug lords, rebel movements and governments. Jacobsson and Naranjo (2004) examine the behavior of drug lords competing in final destination countries and how they react to demand and supply oriented anti-drug policies. By answering this question they are able to explain some stylized trends about drug use, retail drug price and violent crime in the U.S.. However, source countries' illicit behavior is not taken into account implying that supply oriented anti-drug policies are not explicitly modeled. Instead, these policies come into the model as an exogenous parameter. Their results show that by increasing the production cost of the drug (i.e., the wholesale price) is possible to decrease drug use (and distribution networks) and increase the retail drug price. However, they assume that the increase in the production cost may be due to an exogenous increase in supply oriented anti-drug policies. The present paper takes a different approach as here source countries' illicit behavior is explicitly modeled without modeling the illicit behavior in final destination countries. As a result, supply oriented policies are explicitly modeled and demand oriented policies affects the behavior of drug dealers transporting and distributing the drug from source countries to the border of final destination countries.

Brito and Intriligator (1992) study the symbiosis between a guerrilla warfare and the drug trade in a three person differential game (i.e., the guerrilla, the government and the drug lords). The focus in their paper is on the role of drug lords in the guerrilla war where the government can be corrupted, which bears little relation to the questions analyzed here.

However, our modeling of the rebel movement's behavior as supplier of protected territory to the drug lord is akin to that in their paper, and also closely related to that in Collier (1999) and the gang's behavior in Konrad and Skaperdas (1998). The common denominator is that a player extracts rents attached to location (e.g., predation of illegal natural resource exports) and needs to finance an initial phase of growth during which it is unprofitable to operate.

The present paper is structured as follows: the setting up of the model and the presentation of the key results are stated first. In section 3, the anti-drug policies are endogenized, and in the final section a discussion of the results and suggestions for future research are presented.

2 The model

In this section we examine a simultaneous interaction between a monopolistic drug producer and distributor, henceforth referred to as the drug lord, and a rebel movement¹². Both are profit maximizing.

The drug lord's revenues come from selling q units of drugs. It also incurs costs from producing the drug (normalized to one), spending on distribution networks, n , and buying protection from the rebel movement, m . The rebel movement sells protection and receives a fraction of the drug output value. The rebels incur costs in terms of manpower and arms. The aggregate spending by rebels is denoted r . The drug lord and the rebel movement face exogenous government policies in the form of supply oriented anti-drug policies, s , and demand oriented anti-drug policies, d , measured in terms of expenditures on the respective policies (these policies are endogenized in the next section).

Total drug production or the supply of drugs, q^s , has the following technology¹³:

$$q^s = m(s, r)c, \quad (1)$$

where c is the potential drug production and the productivity per acreage, $m(s, r) \in [0, 1]$ is a function of the level of crop eradication, s , and rebel spending, r , with the following properties, $\frac{\partial m(s, r)}{\partial s} < 0$, $\frac{\partial m(s, r)}{\partial r} > 0$, and $\frac{\partial^2 m(s, r)}{\partial^2 s} > 0$. A suitable function for $m(s, r)$ that exhibits the properties mentioned before could for example be a simple version of the classic Tullock rent seeking function¹⁴.

Define the productivity per acreage, $m(s, r)$, as the ratio of the total rebel spending on securing drug lord's potential drug production to the sum of supply oriented anti-drug policies, rebel spending, and a parameter $\rho > 0$ that indicates that, even in the absence of these policies other mechanisms such

¹²Historically, rebel movements in the Andean countries were already living in the territories years later used for drug production by drug lords. As this interaction is a constant struggle between different type of players, modeling it as a vertical integration seems not to be appropriate. Hence, it seems more realistic to model this interaction as a game instead of as a contract between the players. See Thoumi (2003) and Vargas (2003).

¹³In a more general case, $q = m^\lambda(w^\omega c^v)$, where $\lambda, \omega, v > 0$. Input factor w represents workers and other intermediate inputs for the drug production. In the paper, w is assumed to be fixed and equal to one. Then, we will have $q = m^\lambda c^v$ and as it can be proved, the main results of the paper will not change qualitatively.

¹⁴Refers to Tullock (1980). See also Hirshleifer (1991) and Skaperdas (1996) for a discussion on contest success functions. In a more general function of the type $m(s, r) = \frac{\phi(r)}{\phi(r) + \phi(s) + \phi(\rho)}$, when $\frac{\partial \phi(r)}{\partial r}$ is constant, the main results do not change.

as employee thefts, would tend to depreciate the drug output¹⁵. Therefore,

$$m(s, r) = \frac{r}{r + s + \rho}. \quad (2)$$

The marginal cost of producing q units of drug is normalized to one and the amount of drug production paid to the rebel movement is equal to αq . The total drug production that is secured, q , is then sold in final destination countries at the price $p > 0$ (the inverse demand). To simplify the analysis, we assume a linear aggregate demand function for drugs in the final destination country, $q^d = a - bp$, where $b > 0$, and $a > 0$.

However, government demand oriented policies result in that only a fraction $z(d, n)$ of the illegal drug demand is secured by the drug lord. Hence, the equilibrium condition in the final destination countries is $\frac{q^s}{z(d, n)} = q^d$, or in terms of the inverse demand function:

$$p = \frac{a}{b} - \frac{q^s}{bz(d, n)}. \quad (3)$$

As we assume that the illegal market is in equilibrium, we drop the superscripts for demand and supply.

In addition, we assume that the fraction z depends on d and n in the following way,

$$z(d, n) = \frac{n}{n + d + \epsilon}, \quad (4)$$

where $\epsilon > 0$ can be interpreted as those other factors that affect the transaction between a seller and a user. Hence, if the ratio between demand oriented anti-drug policies and the distribution networks increases, a larger share of the market will not be reached by these networks. That is, $\frac{\partial z(d, n)}{\partial d} < 0$ and $\frac{\partial z(d, n)}{\partial n} > 0$. Note that this "distribution" technology exhibits increasing marginal returns to the demand oriented anti-drug policies, i.e. $\frac{\partial^2 z(d, n)}{\partial^2 d} > 0$.

From equations (3) and (4) is clear that an increase in demand oriented anti-drug policies by the government has the direct effect of reducing the drug price. Hence, the drug lord can reduce these losses by spending more on the distribution networks, n .

2.1 The drug lord

Having defined the drug lord's revenues and costs¹⁶, total profits are then:

$$\pi_{dl} = pq - (1 + \alpha)q - n. \quad (5)$$

¹⁵See Konrad and Skaperdas (1998) for more on this functional form.

¹⁶Between the years 1986 and 2000, the average share of the hectares eradicated over the total hectares cultivated was 15%. Therefore, in a long run perspective it seems reasonable

Note that for drug production to be a profitable venture is necessary that the willingness to pay for the first unit, $\frac{a}{b}$, exceeds marginal cost, $(1 + \alpha)$. We assume this to be the case.

The drug lord then chooses distribution networks, n , and the potential drug production, c , given the levels of supply oriented anti-drug policies, s , demand oriented policies, d , and rebel spending, r . Note that while the drug lord pays for protection with part of the drug production it does not have control over how much will be protected as this is a rebel's decision. Therefore, it cannot directly decide over q .

The first order conditions with respect to n and c are¹⁷:

$$\frac{\partial \pi_{dl}}{\partial n} = \frac{(m(s, r)c)^2}{b} \left(\frac{d + \epsilon}{n^2} \right) - 1 \leq 0$$

$$\frac{\partial \pi_{dl}}{\partial c} = m(s, r) \left(\frac{a}{b} - \frac{2m(s, r)c}{bz(d, n)} - (1 + \alpha) \right) \leq 0.$$

In addition, we have the complementary slackness conditions for corner solutions. Solving this equation system for n and c yields:

$$n = \max \left\{ B \sqrt{\frac{d + \epsilon}{b}}, 0 \right\}. \quad (6)$$

$$c(r) = \max \left\{ \frac{B}{m(s, r)}, 0 \right\}. \quad (7)$$

Note that as the drug lord chooses the amount of potential drug production c , total drug production in the market, q , will be equal to B , where:

$$B = \frac{b}{2} \left(\frac{a}{b} - (1 + \alpha) - 2 \sqrt{\frac{d + \epsilon}{b}} \right), \quad (8)$$

which is closely related to the difference between marginal revenue and marginal cost of drug production. If $B \leq 0$, the best the drug lord can do is to close down operations, i.e. to set $n = c = 0$.

In equation (6) the optimal level of distribution networks neither depends on rebel spending, nor on supply oriented policies. This is a consequence of the cost structure faced by the drug lord.

to think that the cost of the hectares eradicated is not an important factor for the drug lord. This assumption makes both the marginal revenue and cost of the potential drug production, c , be proportional to the productivity per acreage, m .

¹⁷If $c > 0$ and $n > 0$ the following second order condition holds $1 \geq \frac{d + \epsilon}{n + d + \epsilon}$.

Expression (7) implies that, for a given rebel spending level, an increase in demand oriented anti-drug policies decreases the potential drug production, whereas an increase in supply oriented policies increases it. Note that the effect of these demand policies is a net effect as distribution networks and potential drug production are interdependent. Moreover, as rebel spending affects the drug lord's best response, any change in rebel spending due to changes in supply and/or demand oriented anti-drug policies will have an *indirect* effect on the potential drug production. Therefore, the net effect of an increase in any anti-drug policy will depend on the direction and level of both the *direct* effects ($\frac{\partial c(r)}{\partial s}$ and/or $\frac{\partial c(r)}{\partial d}$) and the *indirect* effects ($\frac{\partial c(r)}{\partial r} \frac{dr}{ds}$ and/or $\frac{\partial c(r)}{\partial r} \frac{dr}{dd}$).

2.2 The rebel movement

The rebel movement maximizes profits by choosing the level of rebel spending, r , to secure the potential drug production, c . Its revenue comes from the fraction of drug production received from the drug lord and the total cost is equal to r . Hence,

$$\pi_r = \alpha m(s, r) c - r. \quad (9)$$

The first order condition with respect to r is¹⁸:

$$\frac{\partial \pi_r}{\partial r} = \alpha \left(\frac{s + \rho}{(r + s + \rho)^2} \right) c - 1 \leq 0$$

Hence,

$$r(c) = \max \left\{ \sqrt{\alpha c(s + \rho)} - (s + \rho), 0 \right\}. \quad (10)$$

This is the rebel's best response function with respect to the potential drug production, c ¹⁹. Hence, rebel spending is positive if $c > \frac{s+\rho}{\alpha}$, i.e. the level of supply oriented anti-drug policies should not be too high compared to the potential drug production (and relative to the fraction α).

Note that, for a given potential drug production, an increase in demand oriented policies does not affect rebel spending, and an increase in supply oriented policies increases (decreases) it if $c > 4\frac{s+\rho}{\alpha}$ ($c < 4\frac{s+\rho}{\alpha}$). These are the *direct* effects.

¹⁸In the case of $\pi_r = \alpha m(s, r) c^\theta - r c^\nu$ the results in this paper hold qualitatively if $\theta > \nu$. Since protection has increasing returns to scale (once you protect one acreage it is less expensive to protect one more acreage), this is a reasonable assumption.

¹⁹If $r > 0$ the following second order condition holds $\alpha c(s + \rho) \left(\frac{-2}{(r + s + \rho)^3} \right) \leq 0$.

However, both anti-drug policies affect c and indirectly rebel spending through the rebel movement's best response function.

As an increase in the potential drug production increases rebel spending (equation (10)), any change in c from changes in supply and/or demand oriented policies will have an *indirect* effects on rebel spending. Therefore, the net effect of an increase in any anti-drug policy will depend on the direction and level of both the *direct* effects $(\frac{\partial r(c)}{\partial s})^{20}$ and the *indirect* effects $(\frac{\partial r(c)}{\partial c} \frac{dc}{ds})$ and/or $\frac{r(c)}{\partial c} \frac{dc}{dd}$.

Figure 1 illustrates the best responses of the drug lord and the rebel movement and the equilibrium of the game.

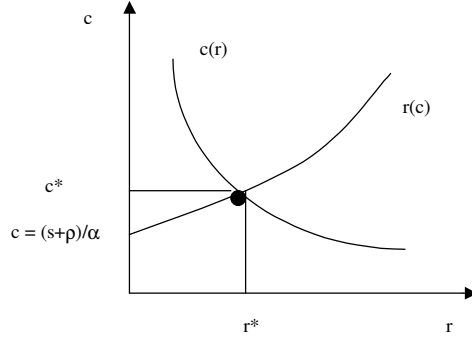


Figure 1. Best response functions for the drug lord and the rebel movement.

2.3 Equilibrium

In the last section we introduced the players' behavior and reaction functions. What comes next is to characterize the equilibrium.

From equation (6) we have that distribution networks in equilibrium are equal to:

$$n^* = \max \left\{ B \sqrt{\frac{d + \epsilon}{b}}, 0 \right\}. \quad (11)$$

Inserting equation (10) into (7), and solving for c^* yields:

$$c^* = \max \left\{ B \left(1 + \frac{1}{-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\alpha B}{s+p}}} \right), 0 \right\}. \quad (12)$$

And if $B \leq 0$, the best the drug lord can do is to set $c^* = 0$, while if $B > 0$, an interior solution is reached.

²⁰Since $\frac{\partial r(c)}{\partial d} = 0$.

Inserting equation (12) into (10) and solving, yields:

$$r^* = \max \left\{ (s + \rho) \left(-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\alpha B}{s + \rho}} \right), 0 \right\}. \quad (13)$$

If $B \leq 0$, the best the rebel movement can do is to set $r^* = 0$, i.e. the drug lord closes down operations, so does the rebel movement. If $B > 0$, an interior solution is reached.

Proposition 1 characterizes the equilibria of the game.

Proposition 1 *If $B > 0$, the interior equilibrium (n^*, c^*, r^*) exists and is unique²¹. If $B \leq 0$, then $n^* = c^* = r^* = 0$, is the unique equilibrium.*

Proof. *See the Appendix.*

Given the equilibrium behavior we can proceed to derive total drug production, drug price, and profits in equilibrium. Using the expressions for rebel spending and potential drug production in equilibrium, and solving for q , yields:

$$q^* = \max\{B, 0\}. \quad (14)$$

Furthermore, inserting equations (11) and (14) into equation (3) yields:

$$p^* = \max \left\{ \frac{1}{2} \left(\frac{a}{b} + (1 + \alpha) \right), 0 \right\}. \quad (15)$$

To conclude, the rebel's and drug lord's profits in equilibrium are:

$$\pi_{dl}^* = \max \left\{ \frac{B^2}{b}, 0 \right\}. \quad (16)$$

$$\pi_r^* = \max\{\alpha B - r^*, 0\}. \quad (17)$$

2.4 Comparative Statics

This section examines the behavior of the variables in equilibrium under changes in the parameters, and particularly in the anti-drug policies.

²¹Evaluating the slopes of both best responses at the equilibrium point we have that for the interior equilibrium to be stable the following condition must hold: $|\frac{\partial c}{\partial r}|_{(r^*, c^*)} > |\frac{\partial r}{\partial c}|_{(r^*, c^*)}$. Calculating the slopes at the equilibrium point we have: $|\frac{1+2\sqrt{\frac{1}{4} + \frac{\alpha B}{(s+\rho)^2}}}{\alpha}| > |\frac{-B}{(s+\rho)(-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\alpha B}{(s+\rho)^2}})^2}|$. Solving this expression we get the following condition: $B > \frac{3}{4} \frac{(s+\rho)}{\alpha}$.

2.4.1 Distribution networks

According to equation (11), distribution networks do not depend on the supply oriented anti-drug policies. This is because the direct effects of $m(s, r)$ on the potential drug production and the distribution networks are offset once the interdependency is taken into account.

From the first order conditions, an increase in demand oriented policies has a non-monotonic effect. First, at low levels of d , distribution networks increase due to the *direct* effect on their marginal revenue. However, as demand oriented policies increase, the *indirect* effect through the decrease in the potential drug production (due to a lower drug price), becomes stronger and ultimately decreases distribution networks.

Moreover, an increase in supply oriented policies does not affect distribution networks. As these policies increase, the negative *direct* effect on distribution networks (decreases their marginal revenue) will be offset by the positive *indirect* effect through the potential drug production (increases it as drug price increases).

Proposition 2 summarizes the comparative statics of equation (11).

Proposition 2 *Total spending on distribution networks, n^* : (i) Increases in the intercept of the demand curve, a . (ii) Decreases in the price sensitivity, b , and the fraction paid for protection, α . (iii) Has an inverse U-shape in demand oriented policies, d : it increases for low d but decreases for high d . (iv) Is not affected by supply oriented policies, s .*

Proof. *Immediate with respect to a, b , and α . See the Appendix for d .*

2.4.2 Potential drug production

In the interior equilibrium (equation (12)), an increase in demand oriented policies decreases the potential drug production. Although the *indirect* effect through the increase in distribution networks is positive the *direct* effect on the potential drug production dominates.

Moreover, an increase in supply oriented policies increases the potential drug production. As distribution networks are not affected by these policies the *indirect* effect comes from the effect on rebel spending. Although this *indirect* effect decreases in supply oriented policies when $c > 4\frac{s+\rho}{\alpha}$ (because an increase in rebel spending decreases the potential drug production), the positive *direct* effect on the potential drug production is stronger.

In addition, if $c = 4\frac{s+\rho}{\alpha}$ the only effect on the potential drug production from the increase in supply oriented policies is the *direct* effect. Therefore,

if the *indirect* effect is negative ($c > 4\frac{s+\rho}{\alpha}$), the positive net effect on the potential drug production will be smaller than if the *indirect* effect was positive ($c < 4\frac{s+\rho}{\alpha}$).

Proposition 3 summarizes the comparative statics of equation (12).

Proposition 3 *Potential drug production, c^* : (i) Increases in the intercept of the demand curve, a , and the supply oriented policies, s . (ii) Decreases in the price sensitivity, b , the demand oriented policies, d , and the fraction paid for protection, α .*

Proof. *Immediate for a, b, s , and d . See Appendix for α .*

2.4.3 Rebel spending

In the interior equilibrium (equation (13)), an increase in demand oriented anti-drug policies decreases rebel spending. As there is no *direct* effect, this decrease comes from the *indirect* effect through the decrease in the potential drug production.

Moreover, an increase in supply oriented policies has a non-monotonic *direct* effect on rebel's spending (at low levels is positive ($c > 4\frac{s+\rho}{\alpha}$) while turning negative at high levels ($c < 4\frac{s+\rho}{\alpha}$)) and a positive *indirect* effect through the potential drug production. As the increase in supply oriented policies increases rebel spending in equilibrium, the positive *indirect* effect is stronger than the negative *direct* effect when $c < 4\frac{s+\rho}{\alpha}$.

In addition, if $c = 4\frac{s+\rho}{\alpha}$ the only effect on rebel spending from the increase in supply oriented policies is the positive *indirect* effect. Therefore, if the *direct* effect is negative ($c < 4\frac{s+\rho}{\alpha}$), the positive net effect on rebel spending will be smaller than if the *indirect* effect was positive ($c > 4\frac{s+\rho}{\alpha}$).

Proposition 4 summarizes the comparative statics of rebel spending.

Proposition 4 *Total spending by rebels, r^* : (i) Increases in the intercept of the demand curve, a , and the supply oriented policies, s . (ii) Decreases in the price sensitivity, b , and the demand oriented policies, d . (iii) Has an inverse U-shape in the fraction paid for protection, α : it increases for low α and decreases for high α .*

Proof. *Immediate with respect to a, b, s and d . See the Appendix for α .*

2.4.4 Total drug production, drug price and profits

According to equation (14) an increase in demand oriented anti-drug policies decreases total drug production as it affects negatively both the potential

drug production (in spite of the increase in distribution networks) and the rebel spending in equilibrium.

Moreover, the net effect of an increase in the supply oriented policies on total drug production is the result of a positive effect on the potential drug production and a negative effect on the share of the potential drug production secured by the rebel movement, $m^*(s, r^*)$. Since these two effects happen to offset each other the supply oriented policies do not affect total drug production in equilibrium. This result comes from the cost structure of the model and in particular from the proportionality of the marginal cost drug production, c , to the productivity per acreage, m .

As we see, neither supply nor demand oriented anti-drug policies affect the drug price in equilibrium (equation (15)). Since supply oriented policies neither affect total drug production nor distribution networks in equilibrium, the effect on price is null. However, demand oriented policies affect non-monotonically the distribution networks and negatively the total drug production in equilibrium. Hence, in equilibrium, $z^*(d^*, i)$ decreases in the same proportion as the decrease in total drug production, leaving the price unchanged.

Proposition 5 sums up the comparative statics about the drug use and drug price in equilibrium.

Proposition 5 *Drugs sold in equilibrium, q^* : (i) Increase in the intercept of the demand curve, a . (ii) Decrease in the price sensitivity, b , the fraction paid for protection, α , and the demand oriented policies, d . (iii) Is not affected by the supply oriented policies, s . Equilibrium drug price, p^* : (iv) Increases in a and α . (v) Decreases in b . (vi) Is neither affected by s nor by d .*

Proof. *Immediate.*

In addition, looking at equations (16) and (17) an increase in demand oriented anti-drug policies reduce both players' profits while an increase in supply oriented policies do not affect the drug lord's profits but decrease the rebel's profits.

Thus far, we have not studied the government's actions. The following section endogenizes the anti-drug policies.

3 Setting anti-drug policies

In this section, we extend the model by endogenizing both the supply and demand oriented anti-drug policies. The purpose is to see in what contexts a government will use any (if not both) of these policies.

We assume that the government, in a previous stage, minimizes a simple loss function. Then, solving by backward induction the government knows the equilibrium values from the simultaneous game between the drug lord and the rebel movement.

Since the U.S. government faces the problem of the amount of drugs sold in a final destination country and the Peruvian's and Colombian's governments have the presence of rebel movements in their territories, we will analyze the following two cases: a government combating the amount of drugs sold and a government wanting to diminish rebels' strength by driving down their profits.

3.1 Combating drug use

As it was shown in Proposition (5), the amount of drugs sold in the final destination country is not affected by supply oriented anti-drug policies. Hence, the only anti-drug policies that are effective in equilibrium are demand oriented. Then, suppose the government chooses to minimize the amount of drugs sold in its country. Then, it needs to trade off the social cost of the drugs sold, given by $\phi_{us}q^*$ ($\phi > 0$ is the marginal social cost), against alternative uses of public funds. Let η_{us} be the shadow price of these public funds ($\eta_{us} > 0$).

The government then chooses spending on demand oriented anti-drug policies, d , to solve the following problem:

$$\min_d L_{us}^* = \phi_{us}q^* + \eta_{us}d = \phi_{us}B + \eta_{us}d.$$

Solving for d yields:

$$d^* = \frac{b\phi_{us}^2}{4\eta_{us}^2} - \epsilon. \quad (18)$$

Hence, the spending on demand oriented policies in equilibrium is positive for a sufficiently small ϵ ²². Obviously, a stronger emphasis on combating the amount of drugs sold, i.e., a higher ϕ_{us} , implies higher spending on demand oriented policies while a higher shadow price of public funds, η_{us} , leads to a lower spending.

Comparing equation (18) with condition $B \leq 0$, we see that if $\eta_{us} \leq \frac{\phi_{us}}{\left(\frac{a}{b} - (1+\alpha)\right)}$, the shadow price of public funds is sufficiently low, then is optimal to set these anti-drug policies so that the drug lord and the rebel movement close down operations. Therefore, the cost of implementing these demand oriented policies is sufficiently high to allow that the drug lord and the rebel movement operate in the market.

²²If $d^* > 0$ the following second order condition holds $\frac{\phi_{us}(d+\epsilon)^{-\frac{3}{2}}\sqrt{b}}{4} \geq 0$.

3.2 Driving down the rebel movement's profits

As both demand and supply oriented anti-drug policies diminish the rebel movement's profits, a government wanting to drive them down will use both anti-drug policies in equilibrium. The government chooses simultaneously the optimal level of both supply and demand oriented anti-drug policies knowing the level of profits by the rebel movement in equilibrium²³.

Therefore, the problem for the government is to,

$$\min_{s,d} L_{col} = \phi_{col}\pi_r^* + \mu_{col}s + \eta_{col}d,$$

where π_r^* is given by equation (17). Solving for s and d we have²⁴:

$$s^* = \alpha Ab \left(\frac{a}{b} - (1 + \alpha) - \frac{\alpha\phi_{col}C}{\eta_{col}} \right) - \rho, \quad (19)$$

where $A = -1 + \sqrt{1 + \frac{1}{\frac{4\mu_{col}}{\phi_{col}} + \frac{4\mu_{col}^2}{\phi_{col}^2}}} > 0$ and $C = \left(1 - \frac{1}{\sqrt{1 + \frac{2}{A}}}\right) > 0$. Hence, $s^* > 0$ for a sufficiently small ρ . The optimal level for the demand oriented anti-drug policies is:

$$d^* = \frac{b\phi_{col}^2\alpha^2C^2}{4\eta_{col}^2} - \epsilon, \quad (20)$$

which is positive for a sufficiently small ϵ .

Comparing equation (20) with condition $B \leq 0$, we see that if $\eta_{col} \leq \frac{\phi_{col}\alpha C}{\left(\frac{a}{b} - (1 + \alpha)\right)}$, the level of demand oriented anti-drug policies in equilibrium is sufficiently high to make the corner solution the equilibrium in the game. As before, the shadow price of public funds allocated to these policies is sufficiently low that the level of these anti-drug policies makes the drug lord and rebel movement to close down operations.

The following Proposition sums up the main finding in this section.

Proposition 6 *Governments aiming to reduce drugs sold will set a positive level of the demand oriented anti-drug policies while not investing in supply oriented policies. However, governments aiming to drive down rebels' profits will set a positive level in both demand and supply oriented anti-drug policies.*

Proof. *Immediate.*

²³The main result in this section is robust to the sequential choice by a government that first decides over demand oriented policies and then on supply side policies. This can be the case if we think that supply oriented policies are mainly focused on source countries and they may take a longer time to be implemented.

²⁴If $s^* > 0$ and $d^* > 0$ the following second order condition holds $\alpha^2 B^2 > 0$. This condition assures the convexity of the loss function.

4 Discussion

By introducing strategic interactions between a drug lord, a rebel movement and a government, the paper gives some possible explanations to the unexpected negative results in the use of demand and supply anti-drug policies in source countries. In spite of increasing investments in both supply and demand oriented anti-drug policies by U.S. authorities, the size of the territories cultivated with coca and opium poppy leaves in the Andean region has not experienced any dramatic change since 1986. However, the number of rebels in the region has shown a positive trend for the past two decades, with a higher growth rate during the 1980's than during the 1990's. On the other hand, wholesale drug prices have been dropping constantly whereas trafficking activities measured as drug seizures and/or the number of trafficking routes towards the U.S. market have been increasing²⁵.

Our model argues that the trend in the territory cultivated with drugs can be consistent with the opposite effects that both the supply and demand oriented anti-drug policies have over the territory. Although supply oriented policies increase this territory, demand oriented policies decrease it. Moreover, the trend in the number of rebels suggests that the increase in supply oriented anti-drug policies could have had stronger effects than the demand oriented policies. The difference in growth rates between the periods 1984-1991 and 1992-2000, can be explained by the fact that in the first period the size of the drug industry could have been small compared to the spending on supply oriented policies, and any increase in these policies could have increased rebel spending even more than if the size of the industry were bigger, as could have been the case during the second period. This happens because the increase in the size of the territory cultivated with drugs is smaller when the level of these policies is small than when it is high. Hence, the rebel movement must increase even more its spending to secure a higher share of drug production.

According to the model, the positive trends in trafficking activities can be consistent with the increase in demand oriented policies only if the level of these policies is not sufficiently high to start a decrease in these activities, which can be the case. More importantly, our model predicts that the increase in both trafficking activities and total drug production may have offset the effect of both anti-drug policies on drug prices.

On the other hand, as the demand and supply for drugs are located in different countries, there are clearly two types of governments facing the prob-

²⁵See United Nations Office of Drugs and Crime (UNDCO), Global Illicit Drug Trade, 1999-2003, and World Drug Report 2000.

lem of the illicit industry. A U.S. type of government that faces the problem of the amount of drugs sold in its country and an Andean countries' type of governments (mainly Peruvian and Colombian) which face the problem of the presence of rebel movements and the existence of the territories cultivated with drugs. By analyzing two different objectives for the government the paper finds that when a government is interested in combating the amount of drugs sold available in its country, its unique and optimal anti-drug policy is to increase demand oriented anti-policies (e.g., disrupting distribution networks). However, if a government is interested in driving down the rebel movement's profits, is optimal to increase both demand and supply oriented anti-drug policies, since both decrease rebel's profits. Therefore, a possible explanation to see positive levels in both anti-drug policies is that behind the use of the anti-drug policies both objectives must be represented.

To sum up, the present paper discusses the effects of both demand and supply oriented policies in source countries when strategic effects between a drug lord and a rebel movement are taking into account. The main conclusion from the paper is that supply oriented policies may have a counterproductive effect on the drug industry as drug lords can react by increasing both the territory cultivated with drugs and the distribution networks resulting in a null effect on drug prices and total drug production. Then, this will encourage the rebel movement to increase its rebel spending (e.g., man power) in source countries. On the other hand, demand oriented policies seem to be more important in combating the amount of drugs sold in final destination countries (and the size of rebel movements in source countries) as they attack directly the size of the market faced by drug lords who can never offset their effects in spite of their partial reaction by increasing their distribution networks. This leads to a decrease in the territory cultivated with drugs and in rebel spending in source countries.

Although this paper represents a step forward in the analysis of the effects of anti-drug policies on drug lords and its connection with rebel's behavior in source countries, it is only a first step to understand this important issue. In a future, can be interesting to look at the effects of market structures in this illicit industry. Moreover, using the results from this paper to know which variables can play a strategic role in this industry (e.g., distribution networks or territory cultivated with drugs), interesting questions to be addressed in an empirical context can be the independency of any anti-drug policy on drug price and/or the irrelevance of supply oriented policies on total drug production (or amount of drugs sold) and wholesale price trends in the U.S. market.

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5 Appendix

5.1 Data

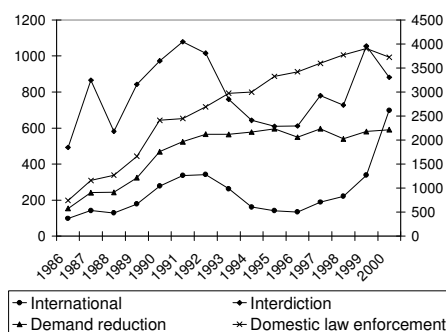


Figure A1. US International, Interdiction, Domestic law enforcement and Demand reduction expenditures (1979 USD, in millions), 1986-2002.

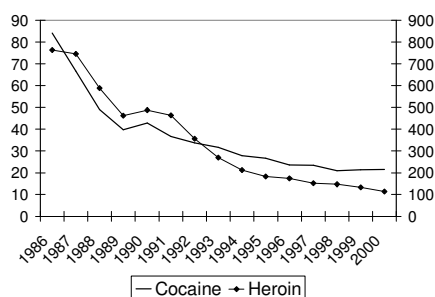


Figure A2. Average cocaine and heroin wholesale prices per pure gram in US, 1986-2001 (1979 USD).

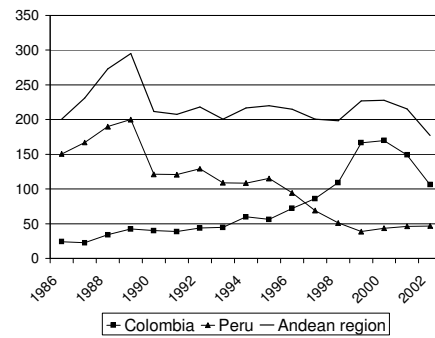


Figure A3. Hectares cultivated with coca and opium poppy leaves (in thousands) 1986-2002.

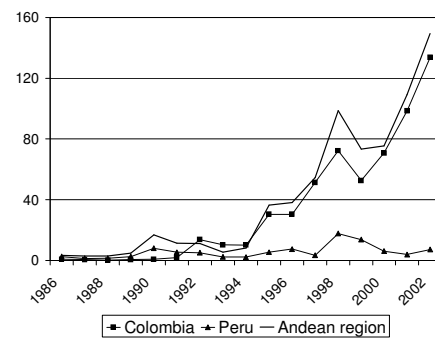


Figure A4. Hectares eradicated of coca and opium poppy leaves (in thousands) 1986-2002.

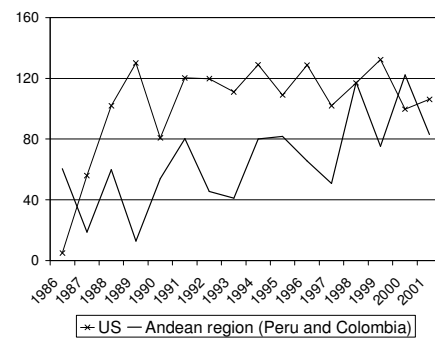


Figure A5. Kilograms of cocaine seized (in thousands) 1986-2001.

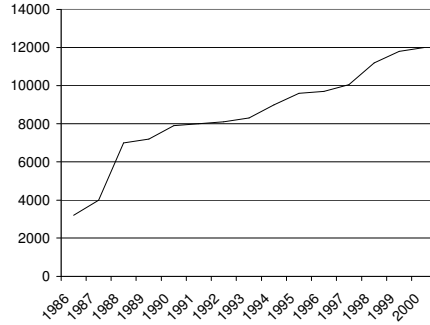


Figure A6. Number of FARC rebels 1986-2000 (Direccion de Inteligencia E.J.C. Folleto Evolucion y Composicion de Grupos Terroristas, Junio 2000).

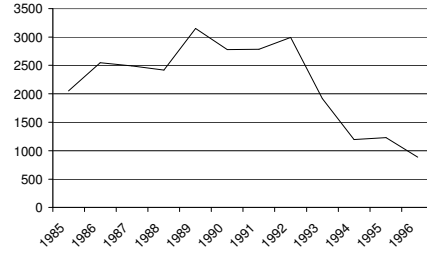


Figure A7. Subversive incidents in Peru, 1985-1996 (Ministry of Interior 1997, Peru).

5.2 Proofs

Proposition (1)

(a) The interior equilibrium: We need to prove that both best response functions cross once and only once. $B > 0$ implies $c(r) > B \forall r > 0$ and $\frac{\partial c(r)}{\partial r} < 0$, given $c(r)$ continuity and differentiability. Therefore, $c(r) \in (\infty, B) \forall r > 0$. On the other hand, the inverse best response function for the rebel movement (i.e., in terms of r), $c_r(r) > \frac{s+\rho}{\alpha} \forall r > 0$ and $\frac{\partial c_r(r)}{\partial r} > 0$, given $c_r(r)$ continuity and differentiability. Therefore, $c_r(r) \in (\frac{s+\rho}{\alpha}, \infty) \forall r > 0$. Since $\frac{s+\rho}{\alpha} < \infty$ and $B < \infty$ then both response functions must cross. Since $\frac{\partial c(r)}{\partial r} < 0$ and $\frac{\partial c_r(r)}{\partial r} > 0$ they cross only once.

(b) The corner solution: If $B \leq 0$ then $c(r) = 0$ and $d = 0$. Moreover, $r(c = 0) = 0$ as $c < \frac{s+\rho}{\alpha}$ (assuming that $s + \rho > 0$ and $\alpha > 0$). Q.E.D.

Proposition (2)

To prove that n^* has a hump-shaped relationship to d for positive values of n^* and d , it is sufficient to show that equation (13) is strictly concave

and achieves a maximum for $d > 0$. First, let us examine the concavity: $\frac{\partial d^{2*}}{\partial^2 d} = -\frac{\sqrt{b}}{8}(\frac{a}{b} - (1 + \alpha))(d + \epsilon)^{-\frac{2}{3}} < 0$. Hence, we have strict concavity as $(\frac{a}{b} - (1 + \alpha)) > 0$ and $\epsilon > 0$. Setting the first derivative of equation (13) with respect to d to zero and solving for d yields: $d^{\max} = \frac{b}{16}(\frac{a}{b} - (1 + \alpha))^2 - \epsilon > 0$ (as $n^* > 0$). Hence, equation (13) peaks for a positive value of d (given a sufficiently small ϵ). Q.E.D.

Proposition (3)

We want to prove that c^* decreases for any value of α . Taking the first derivative of equation (12) with respect to α we have:

$$\frac{\partial c^*}{\partial \alpha} = B \left(\frac{-\frac{1}{2(s+\rho)}(\frac{1}{4} + \frac{\alpha B}{(s+\rho)})^{-\frac{1}{2}}(B - \frac{\alpha b}{2})}{(-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\alpha B}{(s+\rho)}})^2} \right) + \left(1 + \frac{1}{-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{\alpha B}{(s+\rho)}}} \right) \left(-\frac{b}{2} \right) \geq 0.$$

From condition $B > 0$, we know that $\alpha < (\frac{a}{b} - 1) - 2\sqrt{\frac{(d+\epsilon)}{b}}$.

Therefore, we have the following two cases:

(1) If $B - \frac{\alpha b}{2} > 0$ ($\alpha < \frac{1}{2}(\frac{a}{b} - 1) - \sqrt{\frac{(d+\epsilon)}{b}}$) then $\frac{\partial c^*}{\partial \alpha} < 0$.

(2) If $B - \frac{\alpha b}{2} \leq 0$ ($\alpha \geq \frac{1}{2}(\frac{a}{b} - 1) - \sqrt{\frac{(d+\epsilon)}{b}}$) then

$$\frac{(s+\rho)}{b} \left(\frac{1}{2} \left(\frac{a}{b} - 1 \right) - \sqrt{\frac{(d+\epsilon)}{b}} - \frac{3}{2}\alpha \right) - \alpha^3 \geq 0.$$

Because in case (2) $\alpha \geq \frac{1}{2}(\frac{a}{b} - 1) - \sqrt{\frac{(d+\epsilon)}{b}}$ that implies

$$\alpha \geq \frac{2}{3} \left(\frac{1}{2} \left(\frac{a}{b} - 1 \right) - \sqrt{\frac{(d+\epsilon)}{b}} \right) \text{ and } \left(\frac{1}{2} \left(\frac{a}{b} - 1 \right) - \sqrt{\frac{(d+\epsilon)}{b}} - \frac{3}{2}\alpha \right) \leq 0.$$

Therefore, $\frac{(s+\rho)}{b} \left(\frac{1}{2} \left(\frac{a}{b} - 1 \right) - \sqrt{\frac{(d+\epsilon)}{b}} - \frac{3}{2}\alpha \right) - \alpha^3 \leq 0$ and $\frac{\partial c^*}{\partial \alpha} \leq 0$. Q.E.D.

Proposition (4)

To prove that r^* has a hump-shaped relationship to α for positive values of r^* and α it is sufficient to show that equation (11) is strictly concave and achieves a maximum for $\alpha > 0$. First, let us examine the concavity:

$$\frac{\partial^2 r^*}{\partial^2 \alpha} = 2 \left(\frac{\partial B}{\partial \alpha} \right) \left(\frac{s+\rho}{2} \right) \left(\frac{(s+\rho)^2}{4} + \alpha B(s+\rho) \right)^{-\frac{1}{2}} + \left(\frac{(s+\rho)^2}{2} \right) \left(\alpha \left(\frac{\partial B}{\partial \alpha} \right) + B \right)^2 \left(-\frac{1}{2} \left(\frac{(s+\rho)^2}{4} + \alpha B(s+\rho) \right)^{-\frac{3}{2}} \right) < 0.$$

Hence, we have strict concavity, as $B > 0$, and $\rho > 0$. Setting the first derivative of equation (11) with respect to α , to zero, and solving for α yields: $\alpha^{\max} = \frac{1}{2} \left(\frac{a}{b} - 1 \right) - \sqrt{\frac{(d+\epsilon)}{b}} > 0$ (as $r^* > 0$). Hence, equation (11) peaks for a positive value of α . Q.E.D.